Projet de financement de postdoc

Linking Focusing and Automated Theorem Proving (LFAT)

Axe scientifique du GT de rattachement : SciLex

Nom, laboratoire et coordonnées des porteurs :

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Nom du GT impliqué, équipes impliquées : GT UPSCaLe,

- équipe Parsifal, LIX
- équipe Methodes, SAMOVAR

Les membres impliqués sont tous experts en théorie de la démonstration. Ce post-doc permettra de jeter les ponts d'une nouvelle collaboration qui alliera les connaissances des membres de Parsifal sur la focalisation avec celles de Guillaume Burel sur les raffinements de la résolution.

En cherchant à trouver des liens entre différents systèmes de preuve, ce post-doc répond à l'objectif du groupe de travail UPSCaLe de faire communiquer ces systèmes de preuve entre eux. Il permet également d'étendre l'applicabilité de ProofCert, une des solutions proposées pour atteindre cet objectif.

Durée et dates envisagées du contrat : 12 mois, 04/2019 à 03/2020

Niveau de rémunération envisagé (salaire chargé) : 4150€/mois

Scientific project :

Context One of the elements that explain the huge improvement of automated theorem provers in the recent years relies on the use of proof search methods in which the proof search space is very restricted. However, such restrictions should not compromise completeness of the proof search method: given a valid formula, the method should eventually find a proof. These restrictions therefore generally relies on fundamental results. Among techniques used for restricting the proof search space, one can cite, on one hand, ordering and selection in methods based on Robinson's resolution; and, on the other hand, focusing in sequent calculi, which lead to tableaux and inverse methods. Ordering and selection techniques are mainly used in for first-order classical logic, in automated theorem provers implementing a resolution-like calculus [13–15]. Focused search strategies, on the other hand, are used in automated theorem

provers for linear, intuitionistic, or modal logics [4, 6, 11, 12]. Although these techniques seem unrelated, recent works [3, 7] build bridges between them.

Focusing is also used at the heart of the ProofCert project, which aims at providing a universal framework for checking and combining proofs. In particular, ProofCert should be able to gracefully import proofs found by automated theorem provers using selection techniques.

State of the art Ordered resolution with selection has been introduced by Bachmair and Ganzinger [1]; its completeness is proved by means of a model construction.

Chaudhuri [4] described how to apply focusing to forward search strategies such as the inverse method and non-classical logics such as linear logic. Chaudhuri et al. [7] also showed how to explain a number of standard search algorithms such as forward chaining and backchaining in terms of polarities and focusing.

Liang and Miller [10] provides a general framework for focusing in linear, intuitionistic and classical logics.

Burel [3] shows how to link focusing in classical logic with selection in resolution, by slightly generalizing them both. This gives a framework called focusing with selection. Completeness is lost in general, but the provable formulas coincides on the sequent calculus and on the resolution sides.

Within the ProofCert project, resolution proofs can be checked by a kernel built upon a sequent calculus with focusing [8]. Based on this, the tool `Checkers` [9] is able to check proofs coming from automated theorem provers based on resolution such as E-prover. However, the selection mechanism of the prover is not taken into account. Although it is mainly useful for proof search and not proof checking, the selection mechanism may lead to smaller certificates if it is handled correctly by the checker.

The ProofCert framework is also capable of elaborating proof certificates into fully detailed proofs [2]. As such, the tools associated to ProofCert, such as `Checkers`, can be used as flexible means for importing resolution-style proofs into Coq.

Objectives The overall objective of this postdoc proposal is the study of the fruitful links between focusing and automated theorem proving. More precisely, the postdoc fellow should study the following directions:

1. High impact strategies such as ordering and selection are designed for classical resolution, and are missing in non-classical forward search strategies such as the inverse method. Since focusing is easily adapted to such non-classical logics and since it has provided an explanation for these classical strategies, it may be possible to transfer the benefits of ordering and selection to non-classical logics via focusing.

2. The meta-theory of focusing is notoriously complex and is an ideal candidate for rigorous formalization in formal reasoning systems such as Coq or Abella. However, it is a challenge to build a good reusable framework for formalizing focusing in a variety of logics. One promising direction is to use synthetic techniques [5].

3. Focusing with selection is not complete in general. Indeed, it depends on how literals are selected in formulas. Most of the completeness results, either on the resolution side or on the sequent calculus side, are built upon model constructions. It should therefore be studied how these proofs can be ported to the framework of focusing with selection,

and how they are related. These may lead to new proof of completeness in other logics, therefore feeding Objective 1.

Risks and fallback solutions The space of ideas and tasks suggested for this research proposal are large. Not only are two major logics (classical and intuitionistic) proposed but also several proof formats are considered (resolution with and without selection, rewriting, narrowing, sequent calculus, etc) and several tools (proof checkers, proof elaborators, proof kernels, etc). If difficulties in one of these areas are encountered, the work can naturally move to an adjacent topic. Depending on circumstances, this one-year post-doc could be split into two six-months post-docs.

Expected results and products This project should provide new proof methods for logics beyond classical first-order logic. Given the foundational nature of this work, it is also likely that new connections between well established proof domains—particularly, resolution and sequent calculus—can be found and exploited. Finally, this work is already supported by various established proof systems, such as E-prover and Coq, but also with developing and new technologies found within the ProofCert setting. This work should lead to richer integration of existing tools as well as new proof-manipulating tools to build.

Avis du responsable du GT auquel ces travaux sont adossés : Cf. dernière page.

References

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