Quantum Computation
Model and Programming Paradigm

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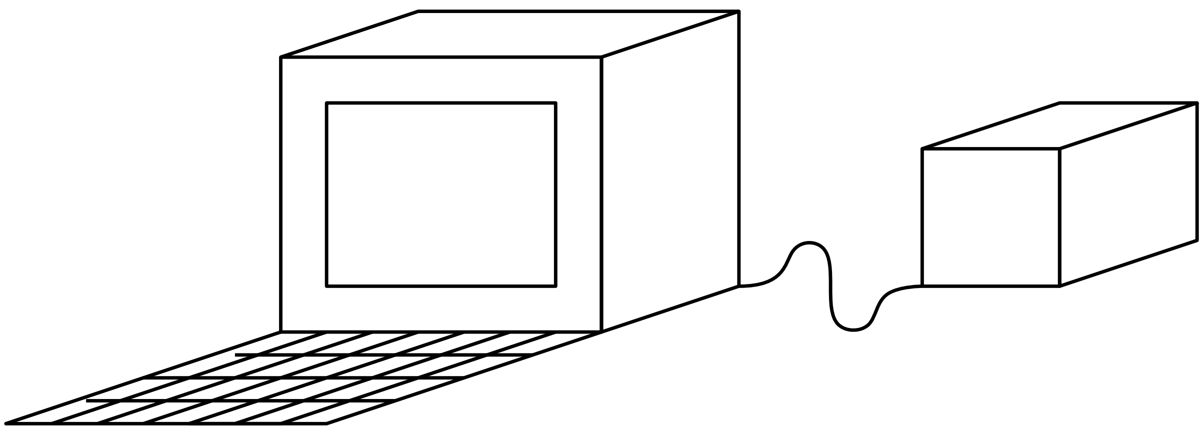
Big Picture: Quantum Computer

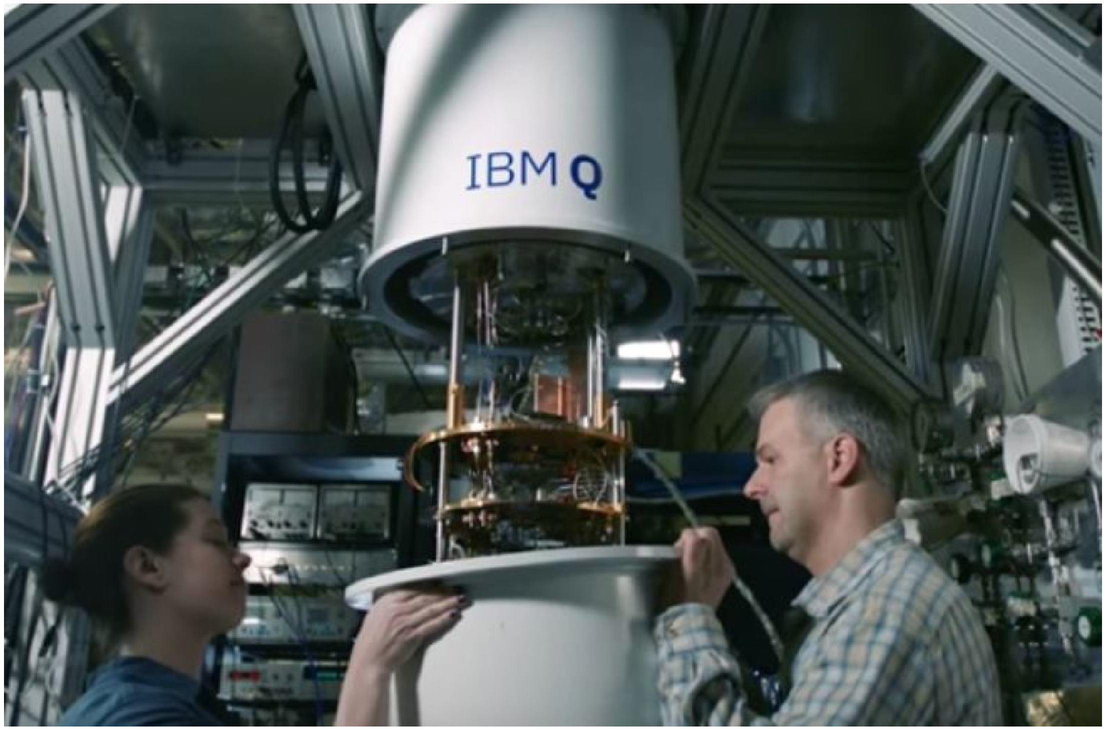
Classical unit = regular computer
Communicates with the coprocessor



Quantum unit = blackbox
Contains a quantum memory

Getting faster algorithms for conventional problems

Big Picture: Quantum Computer

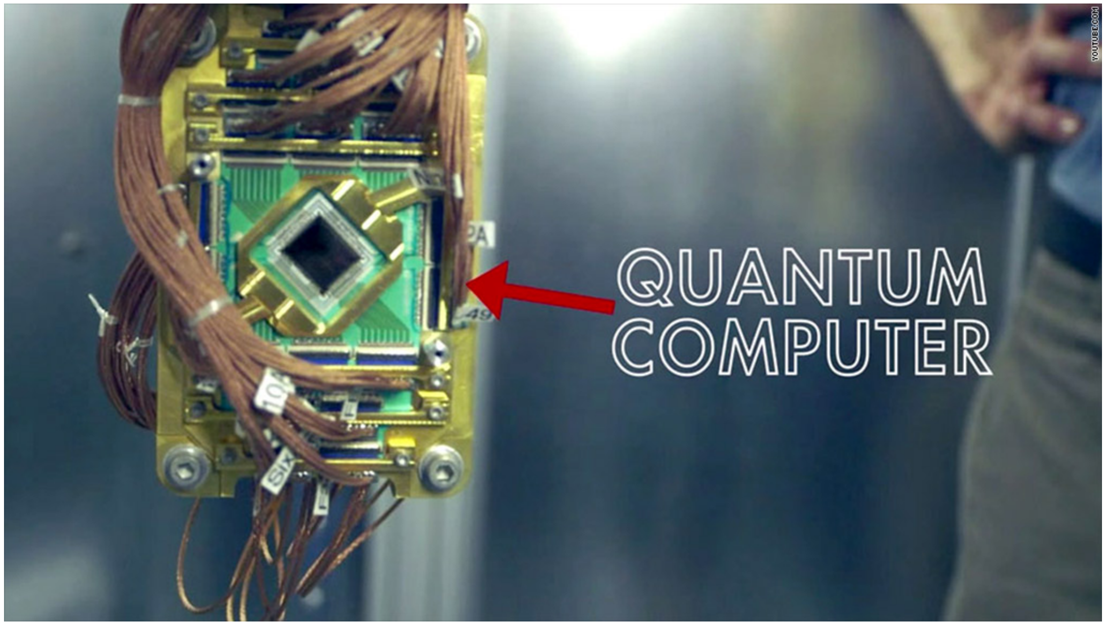


You can access one now !

https://quantumexperience.ng.bluemix.net/qx

Big Picture: Quantum Computer

A small memory-chip inside a big fridge



Big Picture: Quantum Computer

What are quantum algorithms good for?

- factoring !
  - for breaking modern cryptography
- simulating quantum systems !
  - for more efficient molecule distillation procedure
- solving linear systems !
  - for high-performance computing
- solving optimization problems
  - for big learning
- . . . more than 300 algorithms:
  `http://math.nist.gov/quantum/zoo/`

Plan

1. Quantum memory
2. Quantum / Classical interaction
3. Internals of algorithms
4. Coding quantum algorithms
5. The language Quipper
6. A formalization : Proto-Quipper
7. Conclusion

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Quantum memory

A quantum memory with n quantum bits is a complex combination of strings of n bits. E.g. for $n = 3$:

$$
\begin{array}{rl}
& \alpha_0 \cdot \mathsf{000} \\
+ & \alpha_1 \cdot \mathsf{001} \\
+ & \alpha_2 \cdot \mathsf{010} \\
+ & \alpha_3 \cdot \mathsf{011} \\
+ & \alpha_4 \cdot \mathsf{100} \\
+ & \alpha_5 \cdot \mathsf{101} \\
+ & \alpha_6 \cdot \mathsf{110} \\
+ & \alpha_7 \cdot \mathsf{111}
\end{array}
$$

with $|\alpha_0|^2 + |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2 + |\alpha_5|^2 + |\alpha_6|^2 + |\alpha_7|^2 = 1$.

(alike probabilities with complex numbers...)

Quantum memory

The operation one can perform on the memory are of three kinds:

1. Initialization/creation of a new quantum bit in a given *state*:

$$\begin{array}{rl} & \alpha_0 \cdot \mathsf{00} \\ + & \alpha_1 \cdot \mathsf{01} \\ + & \alpha_2 \cdot \mathsf{10} \\ + & \alpha_3 \cdot \mathsf{11} \end{array} \longmapsto \begin{array}{rl} & \alpha_0 \cdot \mathsf{00}\color{blue}{\mathsf{0}} \\ + & \alpha_1 \cdot \mathsf{01}\color{blue}{\mathsf{0}} \\ + & \alpha_2 \cdot \mathsf{10}\color{blue}{\mathsf{0}} \\ + & \alpha_3 \cdot \mathsf{11}\color{blue}{\mathsf{0}} \end{array}$$

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Quantum memory

The operation one can perform on the memory are of three kinds:

2. Measurement. Measuring first qubit:

$$
\begin{array}{ll}
 & \alpha_0 \cdot 00 \\
+ & \alpha_1 \cdot 01 \\
+ & \alpha_2 \cdot 10 \\
+ & \alpha_3 \cdot 11
\end{array}
\longmapsto
\begin{cases}
\begin{array}{ll} & \alpha_0 \cdot 00 \\ + & \alpha_1 \cdot 01 \end{array} & (\text{prob. } |\alpha_0|^2 + |\alpha_1|^2) \\[2em]
\begin{array}{ll} & \alpha_2 \cdot 10 \\ + & \alpha_3 \cdot 11 \end{array} & (\text{prob. } |\alpha_2|^2 + |\alpha_3|^2)
\end{cases}
$$

modulo renormalization.

Quantum memory

The operation one can perform on the memory are of three kinds:

2. Measurement. Measuring second qubit:

$$
\begin{array}{ll}
\alpha_0 \cdot 00 & \\
& + \quad \alpha_1 \cdot 01 \\
+ \quad \alpha_2 \cdot 10 & \\
& + \quad \alpha_3 \cdot 11
\end{array}
\longmapsto
\begin{cases}
\begin{array}{l} \alpha_0 \cdot 00 \\ + \quad \alpha_2 \cdot 10 \end{array} & (\text{prob. } |\alpha_0|^2 + |\alpha_2|^2) \\[2em]
\begin{array}{l} \alpha_1 \cdot 01 \\ + \quad \alpha_3 \cdot 11 \end{array} & (\text{prob. } |\alpha_1|^2 + |\alpha_3|^2)
\end{cases}
$$

modulo renormalization.

Quantum memory

The operation one can perform on the memory are of three kinds:

3. Unitary operations. Linear maps
   - preserving norms,
   - preserving orthogonality,
   - reversible.

   E.g. the N-gate on one quantum bit (flip). On the first qubit:

$$
\begin{array}{rl}
 & \alpha_0 \cdot 00 \\
+ & \alpha_1 \cdot 01 \\
+ & \alpha_2 \cdot 10 \\
+ & \alpha_3 \cdot 11
\end{array}
\longmapsto
\begin{array}{rl}
 & \alpha_0 \cdot 10 \\
+ & \alpha_1 \cdot 11 \\
+ & \alpha_2 \cdot 00 \\
+ & \alpha_3 \cdot 01
\end{array}
$$

Quantum memory

The operation one can perform on the memory are of three kinds:

3. Unitary operations.

   E.g. the Hadamard gate on one quantum bit. Sends

$$\begin{array}{rclcl} \mathtt{0} & \longmapsto & \frac{\sqrt{2}}{2}\cdot\mathtt{0} & + & \frac{\sqrt{2}}{2}\cdot\mathtt{1} \\ \mathtt{1} & \longmapsto & \frac{\sqrt{2}}{2}\cdot\mathtt{0} & - & \frac{\sqrt{2}}{2}\cdot\mathtt{1} \end{array}$$

   When applied on the first qubit:

$$\begin{array}{rl} & \alpha_0 \cdot \mathtt{01} \\ + & \alpha_1 \cdot \mathtt{10} \end{array} \longmapsto \begin{array}{rl} & \alpha_0 \cdot \left(\frac{\sqrt{2}}{2}\cdot\mathtt{01} + \frac{\sqrt{2}}{2}\cdot\mathtt{11}\right) \\ + & \alpha_1 \cdot \left(\frac{\sqrt{2}}{2}\cdot\mathtt{00} - \frac{\sqrt{2}}{2}\cdot\mathtt{10}\right) \end{array}$$

Quantum memory

The operation one can perform on the memory are of three kinds:

3. Unitary operations.

   E.g. the Hadamard gate on one quantum bit. Sends

$$
\begin{array}{ccccc}
0 & \longmapsto & \frac{\sqrt{2}}{2}\cdot 0 & + & \frac{\sqrt{2}}{2}\cdot 1 \\
1 & \longmapsto & \frac{\sqrt{2}}{2}\cdot 0 & - & \frac{\sqrt{2}}{2}\cdot 1
\end{array}
$$

   When applied on the first qubit:

$$
\begin{array}{c} \alpha_0 \cdot 01 \\ + \quad \alpha_1 \cdot 10 \end{array}
\longmapsto
\begin{array}{cl}
 & \alpha_0 \frac{\sqrt{2}}{2} \cdot 01 \\
+ & \alpha_0 \frac{\sqrt{2}}{2} \cdot 11 \\
+ & \alpha_1 \frac{\sqrt{2}}{2} \cdot 00 \\
+ & \alpha_1 \frac{\sqrt{2}}{2} \cdot 10
\end{array}
$$

Quantum memory

The operation one can perform on the memory are of three kinds:

3. Unitary operations.

   They can create superposition...

$$\mathtt{1100} \quad \longmapsto \quad \begin{aligned} & \tfrac{\sqrt{2}}{2} \cdot \mathtt{1100} \\ + \quad & \tfrac{\sqrt{2}}{2} \cdot \mathtt{1110} \end{aligned}$$

   ...or remove it

$$\begin{aligned} & \tfrac{\sqrt{2}}{2} \cdot \mathtt{1100} \\ + \quad & \tfrac{\sqrt{2}}{2} \cdot \mathtt{1110} \end{aligned} \quad \longmapsto \quad \mathtt{1100}$$

Quantum memory

The operation one can perform on the memory are of three kinds:

3. Unitary operations.

   They can simulate classical operations:

   - Bit-flip (N-gate).
   - Tests (Controlled operations). E.g. Controlled-not. Second qubit is controlling:

$$
\begin{array}{rl}
 & \alpha_0 \cdot 00 \\
+ & \alpha_1 \cdot 01 \\
+ & \alpha_2 \cdot 10 \\
+ & \alpha_3 \cdot 11
\end{array}
\longmapsto
\begin{array}{rl}
 & \alpha_0 \cdot 00 \\
+ & \alpha_1 \cdot 11 \\
+ & \alpha_2 \cdot 10 \\
+ & \alpha_3 \cdot 01
\end{array}
=
\begin{array}{rl}
 & \alpha_0 \cdot 00 \\
+ & \alpha_3 \cdot 01 \\
+ & \alpha_2 \cdot 10 \\
+ & \alpha_1 \cdot 11
\end{array}
$$

Quantum memory

The co-processor has an internal (quantum) memory.

- Classical data can transparently flow in.
- Internal operations are local.
- Retrieval of quantum information is probablistic and modify the global state.

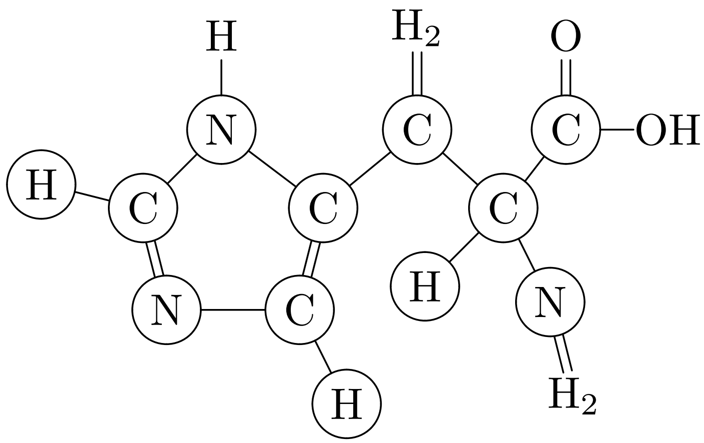
In particular:

- The quantum memory has to be permanent.
- To act on quantum memory, classical operations have to lifted.
- This is potentially expensive.

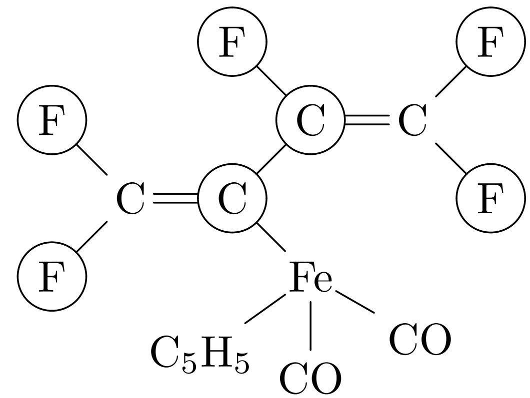
Quantum memory: hardware

Quantum data: encoded on the state of quantum particles.

- E.g. nucleus of an atom:



The histidine as a 12-qubit memory.



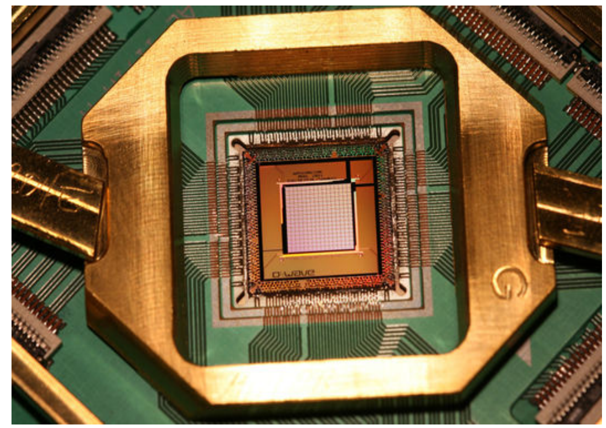
The perfluorobutadienyl iron complex as a 7-qubit memory.

- E.g. Photon polarization.
- E.g. Electrons (in superconducting devices)...

Problems to overcome: Scalability, decoherence.

Nonetheless, we are already post-quantum...

Quantum memory: hardware



Many experts predict a quantum computer capable of effectively breaking public key cryptography within [a few decades], and therefore NSA believes it is important to address that concern.

https://www.iad.gov/iad/library/ia-guidance/ia-solutions-for-classified/algorithm-guidance/cnsa-suite-and-quantum-computing-faq.cfm

Google

Microsoft

Atos

Bull

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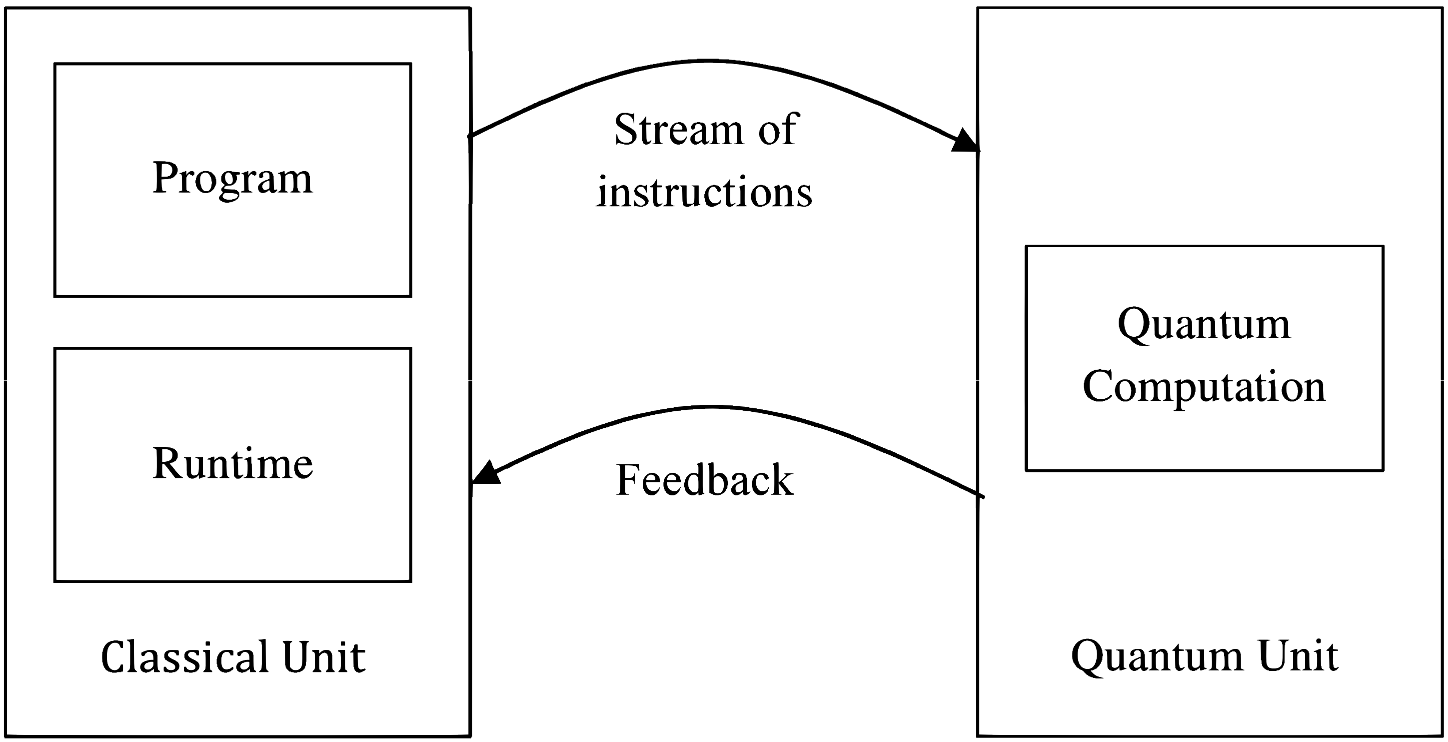
Quantum / Classical interaction

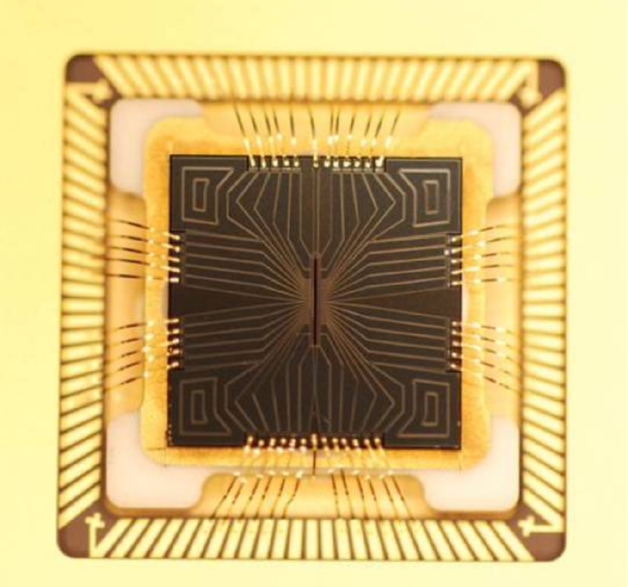
Typical execution flow:



Quantum / Classical interaction

Stream of instructions

- Local actions on one (or two) qubit(s) at a time
- Limited moving of qubits
- No copying



(ion trap)

Quantum / Classical interaction

Stream of instructions

- Local actions on one (or two) qubit(s) at a time
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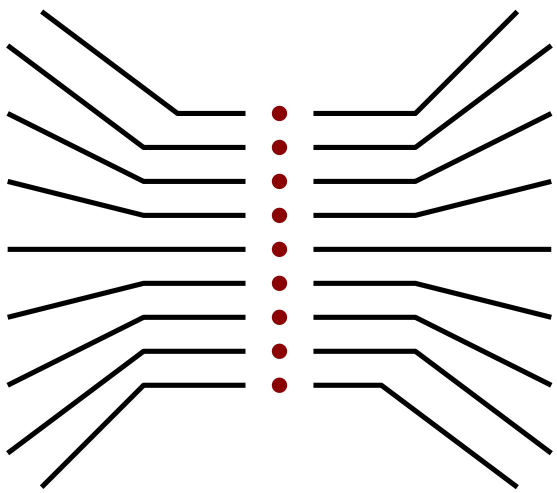
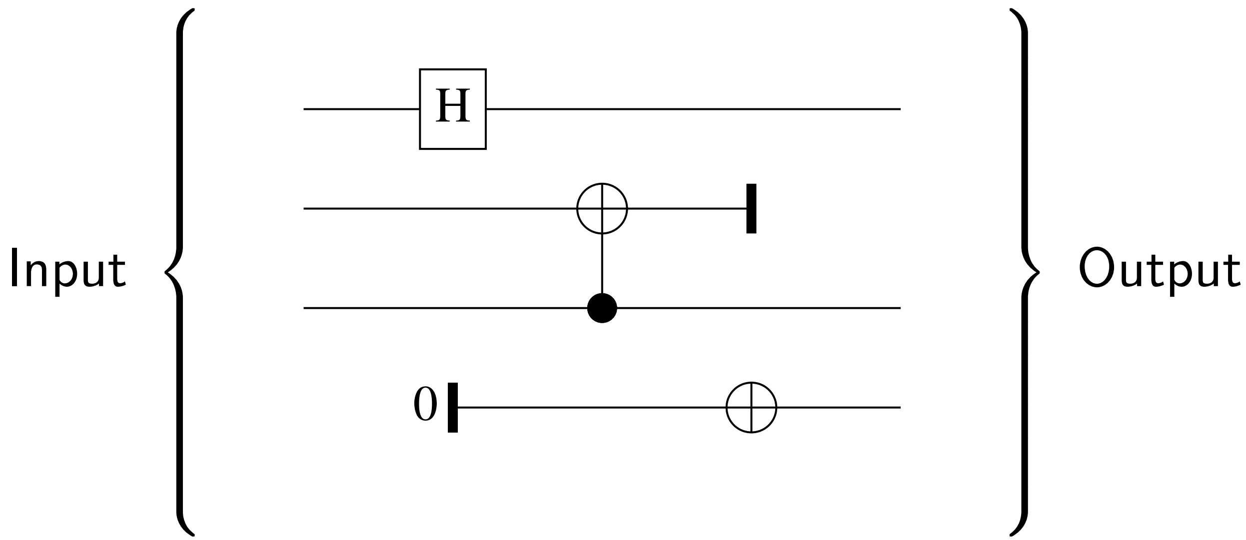


dots $\equiv$ ions $\equiv$ qubits   action $\equiv$ pulses through wires

Quantum / Classical interaction

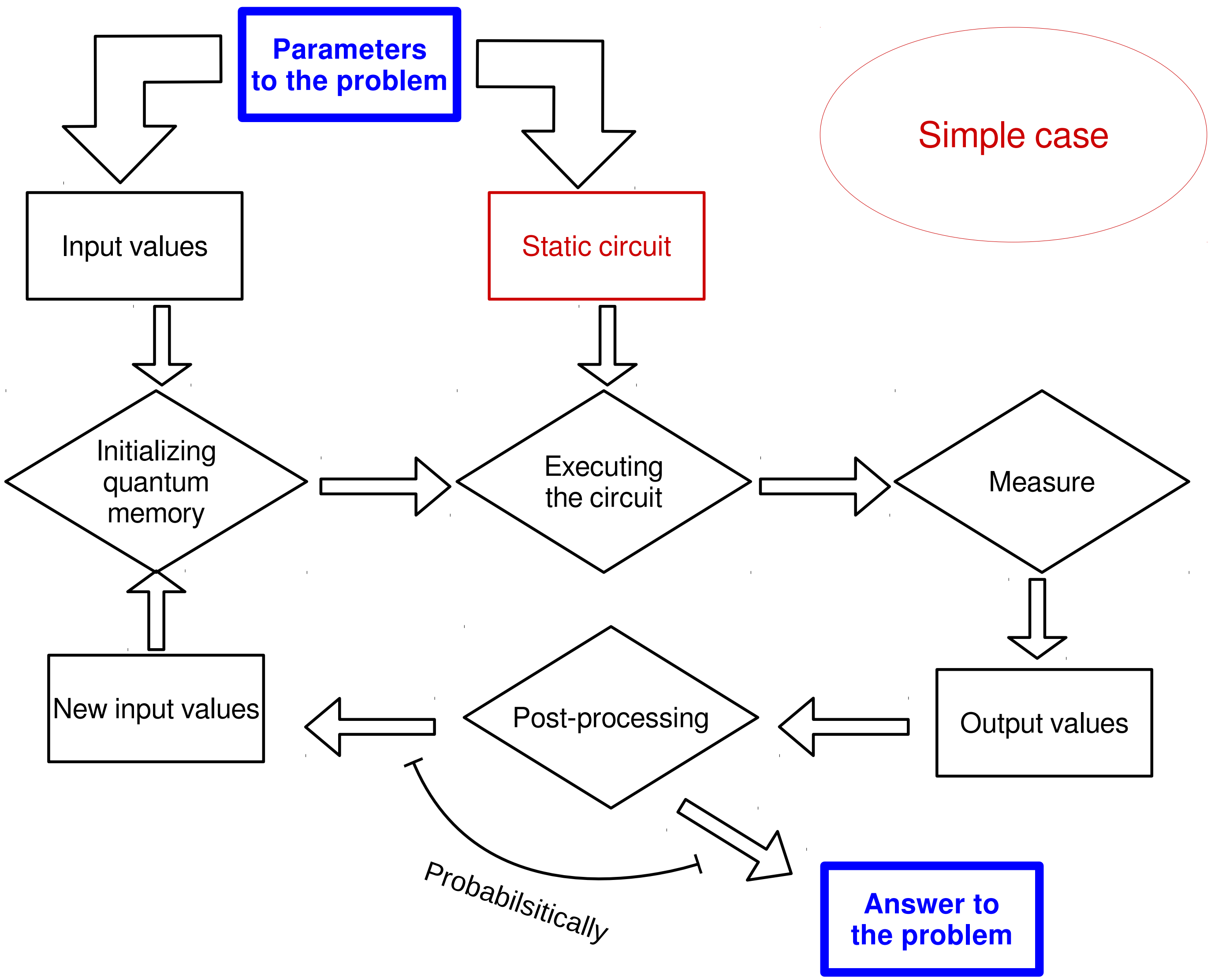
Stream of instructions

- Series of elementary actions applied on the quantum memory
- Summarized with a quantum circuit.
- wire $\equiv$ qubit, box $\equiv$ action, time flows left-to-right



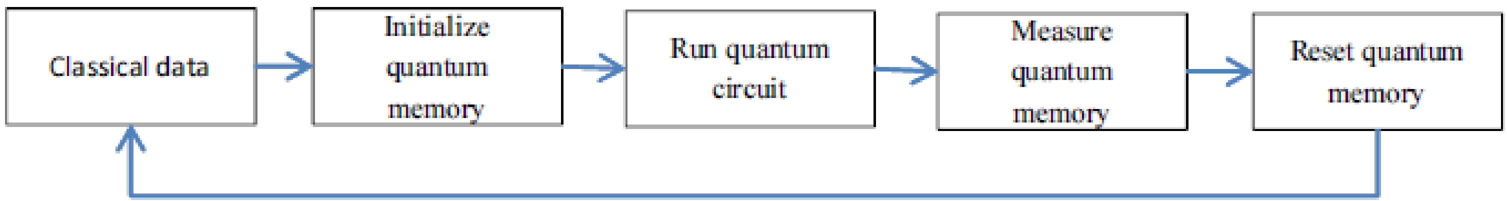
No "quantum loop" or "conditional escape".

Quantum / Classical interaction

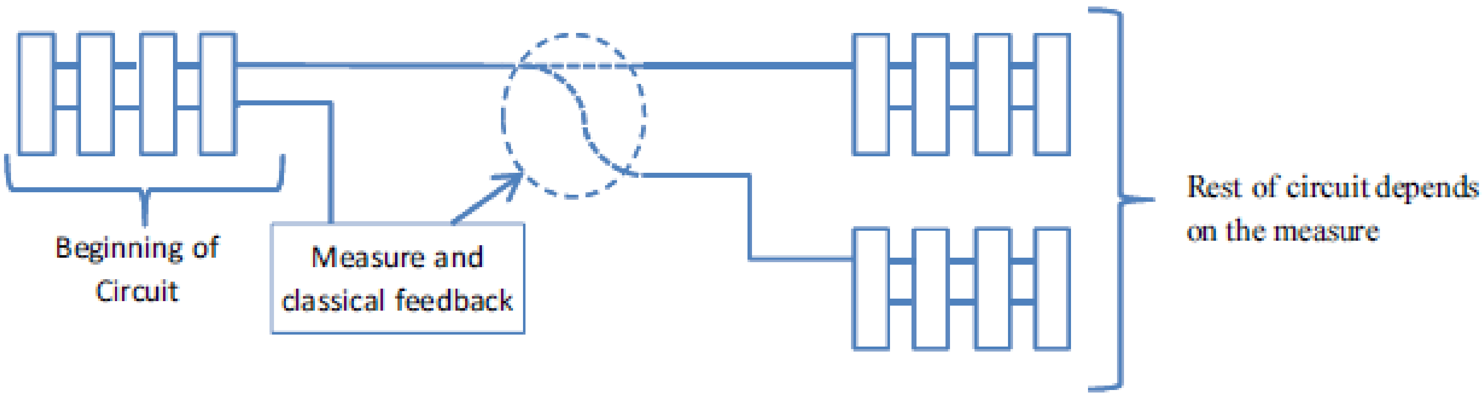


Quantum / Classical interaction

Some algorithms follow a simple scheme



Others are following a more adaptative scheme:



This is where quantum circuits differ from hardware design.

One cannot draw a quantum circuit once and for all.

Quantum / Classical interaction

A sound model of computation:

Interaction with the quantum memory seen as an I/O side effect

```
Circc a := Empty a
         | Write Gate (Circ a)
         | Read Wire (Bool -> (Circ a))
```

- Output: emit gates to the co-processor
- Input: emit a read even to the co-processor, with a call-back function

Representing circuits

- static circuits: lists of gates
- dynamic circuits: trees of gates.

Quantum / Classical interaction

Moral

- Distinction parameter / input
- Circuits might be dynamically generated
- Parameters = govern the shape and size of the circuit
- Model of computation : specialized I/O side-effect

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Internals of algorithms

The techniques used to described quantum algorithms are diverse.

1. Quantum primitives.
   - Phase estimation.

     Assuming $\omega = \mathtt{0.xy}$, we want

$$\begin{array}{rl} & \rho_0(e^{2\pi i\mathtt{xy}})^0 \cdot \mathsf{00} \\ + & \rho_1(e^{2\pi i\mathtt{xy}})^1 \cdot \mathsf{01} \\ + & \rho_2(e^{2\pi i\mathtt{xy}})^2 \cdot \mathsf{10} \\ + & \rho_3(e^{2\pi i\mathtt{xy}})^3 \cdot \mathsf{11} \end{array} \quad \longmapsto \quad 1 \cdot \mathtt{xy}$$

Moving information from coefficients to basis vectors

Internals of algorithms

The techniques used to described quantum algorithms are diverse.

1. Quantum primitives.
   - Phase estimation.
   - Amplitude amplification.

     Qubit 3 in state 1 means good.

$$\begin{array}{cl} & \rho_0 e^{i\phi_0} \cdot 000 \\ + & \rho_1 e^{i\phi_1} \cdot 011 \\ + & \rho_2 e^{i\phi_2} \cdot 100 \\ + & \rho_3 e^{i\phi_3} \cdot 110 \end{array} \longmapsto \begin{array}{cl} & {\scriptstyle \rho_0} e^{i\phi_0} \cdot 000 \\ + & \boldsymbol{\rho_1} e^{i\phi_1} \cdot 011 \\ + & {\scriptstyle \rho_2} e^{i\phi_2} \cdot 100 \\ + & {\scriptstyle \rho_3} e^{i\phi_3} \cdot 110 \end{array}$$

Increasing the probability of measuring the "good" states

Internals of algorithms

The techniques used to described quantum algorithms are diverse.

1. Quantum primitives.
   - Phase estimation.
   - Amplitude amplification.
   - Quantum walk.

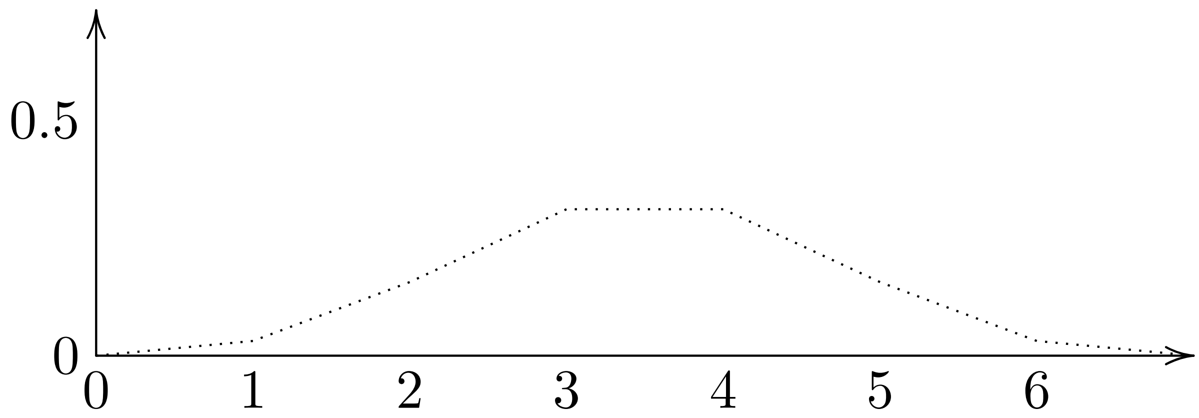
$$\begin{tikzcd}
0 \arrow[r] & 1 \arrow[r] & 2 \arrow[r] & 3 \arrow[r] & 4 \arrow[r] & 5 \arrow[r] & 6 \arrow[r] & 7 \arrow[d] \\
15 \arrow[u] & 14 \arrow[l] & 13 \arrow[l] & 12 \arrow[l] & 11 \arrow[l] & 10 \arrow[l] & 9 \arrow[l] & 8 \arrow[l]
\end{tikzcd}$$

Internals of algorithms

The techniques used to described quantum algorithms are diverse.

1. Quantum primitives.
   - Phase estimation.
   - Amplitude amplification.
   - Quantum walk.

     After 5 steps of a probabilistic walk:



Internals of algorithms

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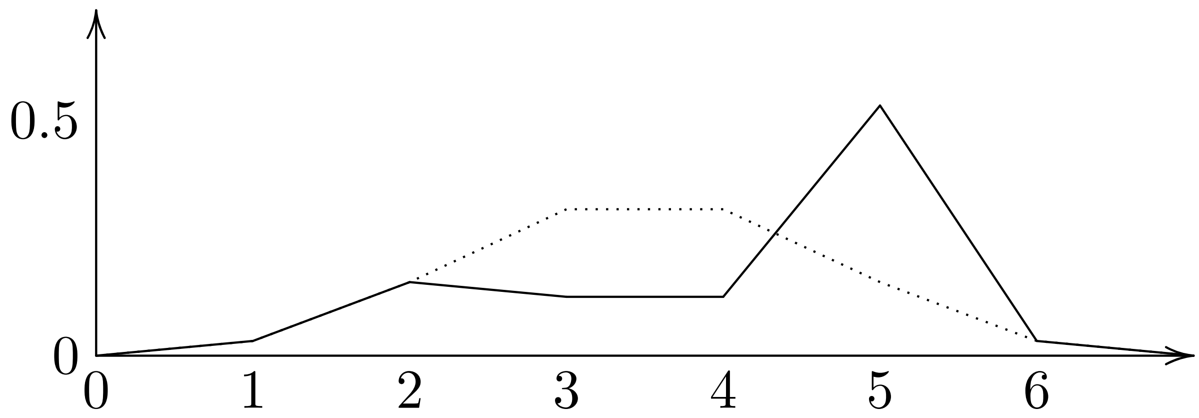


Internals of algorithms

The techniques used to described quantum algorithms are diverse.

2. Oracles.

   - Take a classical function $f : \mathtt{Bool}^n \to \mathtt{Bool}^m$.
   - Construct

$$\begin{array}{rccc} \overline{f}: & \mathtt{Bool}^{n+m} & \longrightarrow & \mathtt{Bool}^{n+m} \\ & (x,y) & \longmapsto & (x, y \oplus f(x)) \end{array}$$

   - Build the unitary U_f acting on $n+m$ qubits computing $\overline{f}$.

Internals of algorithms

The techniques used to described quantum algorithms are diverse.

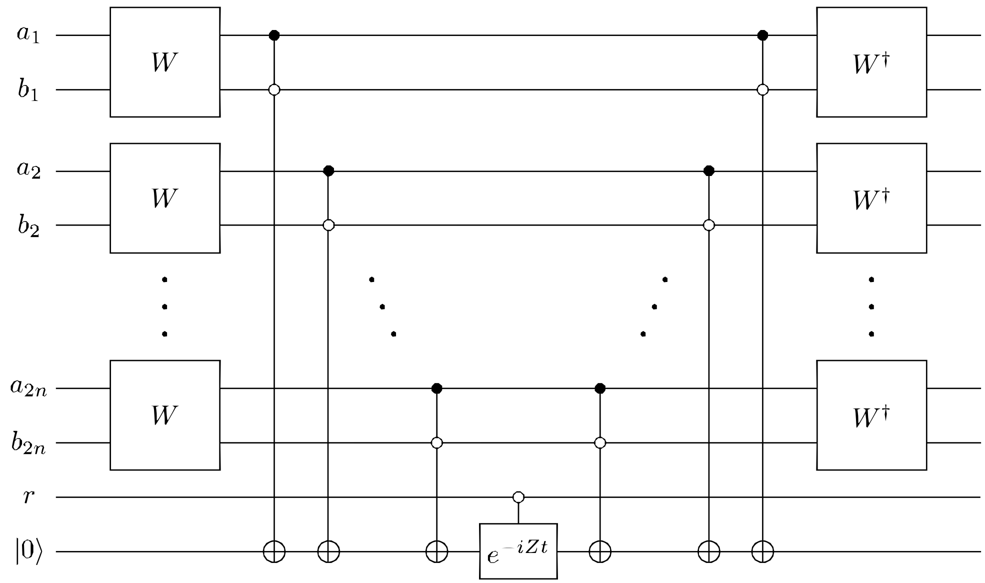
2. Oracles, in real life

```
calcRweights y nx ny lx ly k theta phi =
 let (xc',yc') = edgetoxy y nx ny in
 let xc = (xc'-1.0)*lx - ((fromIntegral nx)-1.0)*lx/2.0 in
 let yc = (yc'-1.0)*ly - ((fromIntegral ny)-1.0)*ly/2.0 in
 let (xg,yg) = itoxy y nx ny in
 if (xg == nx) then
     let i = (mkPolar ly (k*xc*(cos phi)))*(mkPolar 1.0 (k*yc*(sin phi)))*
             ((sinc (k*ly*(sin phi)/2.0))+0.0) in
     let r = ( cos(phi)+k*lx )*((cos (theta - phi))/lx+0.0) in i*r
 else if (xg==2*nx-1) then
     let i = (mkPolar ly (k*xc*cos(phi)))*(mkPolar 1.0 (k*yc*sin(phi)))*
             ((sinc (k*ly*sin(phi)/2.0))+0.0) in
     let r = ( cos(phi)+(- k*lx))*((cos (theta - phi))/lx+0.0) in i*r
 else if ( (yg==1) and (xg<nx) ) then
     let i = (mkPolar lx (k*yc*sin(phi)))*(mkPolar 1.0 (k*xc*cos(phi)))*
             ((sinc (k*lx*(cos phi)/2.0))+0.0) in
     let r = ( (- sin phi)+k*ly )*((cos(theta - phi))/ly+0.0) in i*r
 else if ( (yg==ny) and (xg<nx) ) then
     let i = (mkPolar lx (k*yc*sin(phi)))*(mkPolar 1.0 (k*xc*cos(phi)))*
             ((sinc (k*lx*(cos phi)/2.0))+0.0) in
     let r = ( (- sin phi)+(- k*ly) )*((cos(theta - phi)/ly)+0.0) in i*r
 else 0.0+0.0
```

Internals of algorithms

The techniques used to described quantum algorithms are diverse.

3. Blocks of loosely-defined low-level circuits.



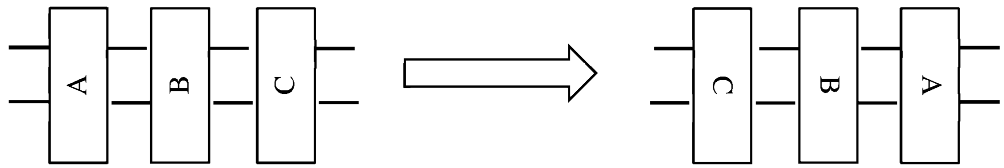
- This is not a formal specification!
- Notion of "box"
- Size of the circuit depends on parameters

Internals of algorithms

The techniques used to described quantum algorithms are diverse.

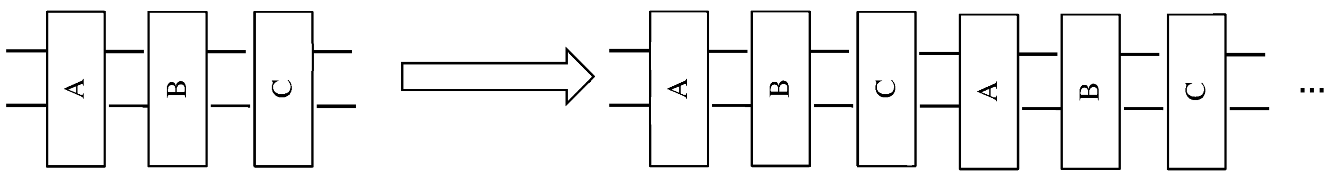
4. High-level operations on circuit:

   - Circuit inversion.



     (the circuit needs to be reversible. . .)

   - Repetition of the same circuit.



     (needs to have the same input and output arity. . .)

   - Controlling of circuits

Internals of algorithms

The techniques used to described quantum algorithms are diverse.

5. Classical processing.
    - Generating the circuit...
    - Computing the input to the circuit.
    - Processing classical feedback in the middle of the computation.
    - Analyzing the final answer (and possibly starting over).

Internals of algorithms

Summary

- Need of automation for oracle generation
- Distinction parameter / input
- Circuits as inputs to other circuits
- Regularity with respect to the size of the input
- Circuit construction:
  - Using circuit combinators: Inversion, repetition, control, *etc*
  - Procedural
- Lots of classical processing!

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Coding algorithms

A very recent topic

- From complexity analysis to concrete circuits
- No machine yet, but
  - Resource analysis
  - Optimization
  - Emulation
- Scalable languages: in the last 5 years
  - Python's libraries/DSL: Project-Q, QISKit, *etc*
  - Liqui$|\rangle$, Q# (Microsoft)
  - Quipper, QWIRE (academic)

Coding algorithms

Imperative programming and the quantum I/O

- Input/Output "as usual": with commands
- Measurement returns a boolean (probabilistically)
- If well-behaved, provides high-level circuit operations
- Example with Project-Q:

```
def circuit(q1,q2):
    H | q1
    with Control(q1):
      X | q2
    x = Measure | q1
    eng.flush()
    if x:
      Y | q2
    else:
      Z | q2
```

Coding algorithms

Functional programming and the quantum I/O

- Monadic approach to encapsulate I/O
- Inside the monad: quantum operations
- Outside the monad: classical operations and circuit manipulation
- Qubits only live inside the monad

Coding algorithms

Dealing with run-time errors

- Imperative-style: Quantum I/O is a memory mapping
  - $\rightarrow$ Type-systems based on separation logic should work
  - Hoare logic or Contracts
- Functional-style:
  - Non-duplicable quantum data: linear type system
  - Dependent-types

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The Language Quipper

- Embedded language in Haskell
- Logical description of hierarchical circuits
- Well-founded monadic semantics. Allow to mix two paradigms
  - Procedural : describing low-level circuits
  - Declarative : describing high-level operation
- Parameter/input distinction
  - Parameter : determine the shape of the circuit
  - Input : determine what goes in the wires
- $\cdots$

The Language Quipper

A function in Quipper is a map

`A -> Circ B`

- Input something of type `A`
- Output something of type `B`
- As a side effect, generate a circuit snippet

Or

- Input a value of type `A`
- Output a "computation" of type `Circ B`

Families of circuits

- represented with lists, e.g. `[Qubit] -> Circ [Qubit]`

The Language Quipper

New base type : `Qubit` $\equiv$ wire

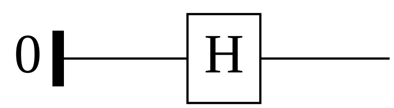
Building blocks

- `qinit :: Bool -> Circ Qubit`
- `qdiscard :: Qubit -> Circ ()`
- `hadamard :: Qubit -> Circ Qubit`
- `hadamard_at :: Qubit -> Circ ()`

Composition of functions $\equiv$ composition of circuits

$$\texttt{Bool} \xrightarrow{\texttt{qinit}} \texttt{Circ Qubit}$$

$$\texttt{Qubit} \xrightarrow{\texttt{hadamard}} \texttt{Circ Qubit}$$

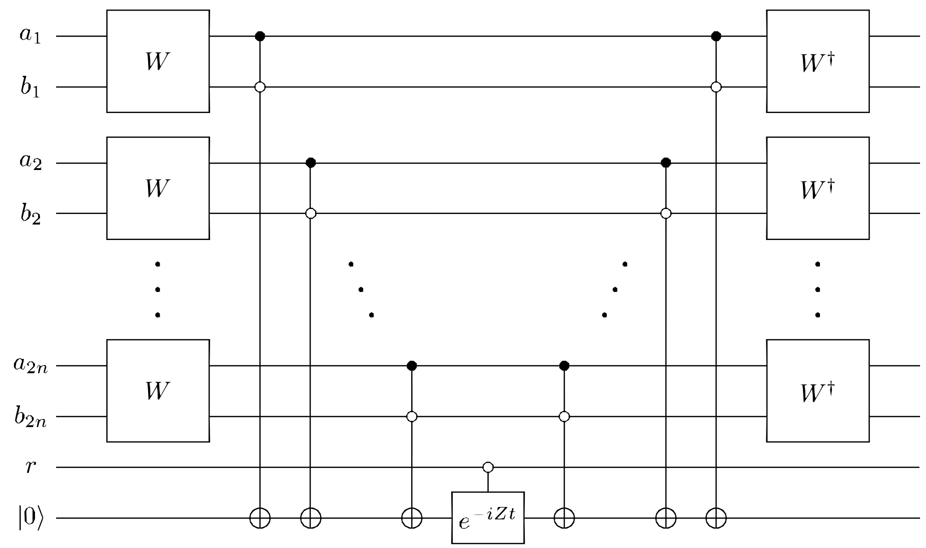


High-level circuit combinators

- `controlled :: Circ a -> Qubit -> Circ a`
- `inverse :: (a -> Circ b) -> b -> Circ a`

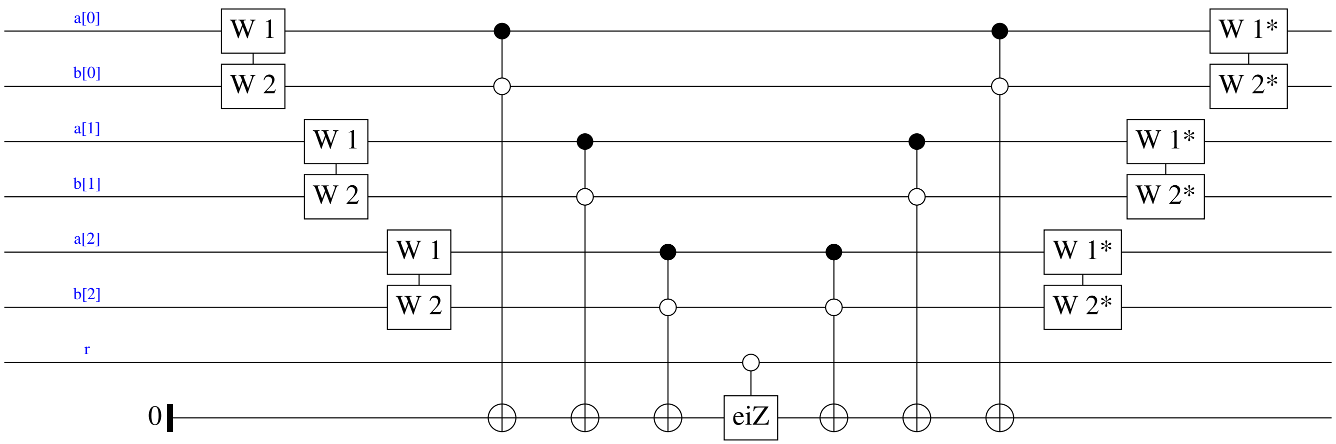
Coding quantum algorithms: Quipper

```
import Quipper

w :: (Qubit,Qubit) -> Circ (Qubit,Qubit)
w = named_gate "W"

toffoli :: Qubit -> (Qubit,Qubit) -> Circ Qubit
toffoli d (x,y) =
  qnot d `controlled` x .==. 1 .&&. y .==. 0

eiz_at :: Qubit -> Qubit -> Circ ()
eiz_at d r =
  named_gate_at "eiZ" d `controlled` r .==. 0

circ :: [(Qubit,Qubit)] -> Qubit -> Circ ()
circ ws r = do
  label (unzip ws,r) (("a","b"),"r")
  d <- qinit 0
  mapM_ w ws
  mapM_ (toffoli d) ws
  eiz_at d r
  mapM_ (toffoli d) (reverse ws)
  mapM_ (reverse_generic w) (reverse ws)
  return ()

main = print_generic EPS circ (replicate 3 (qubit,qubit)) qubit
```



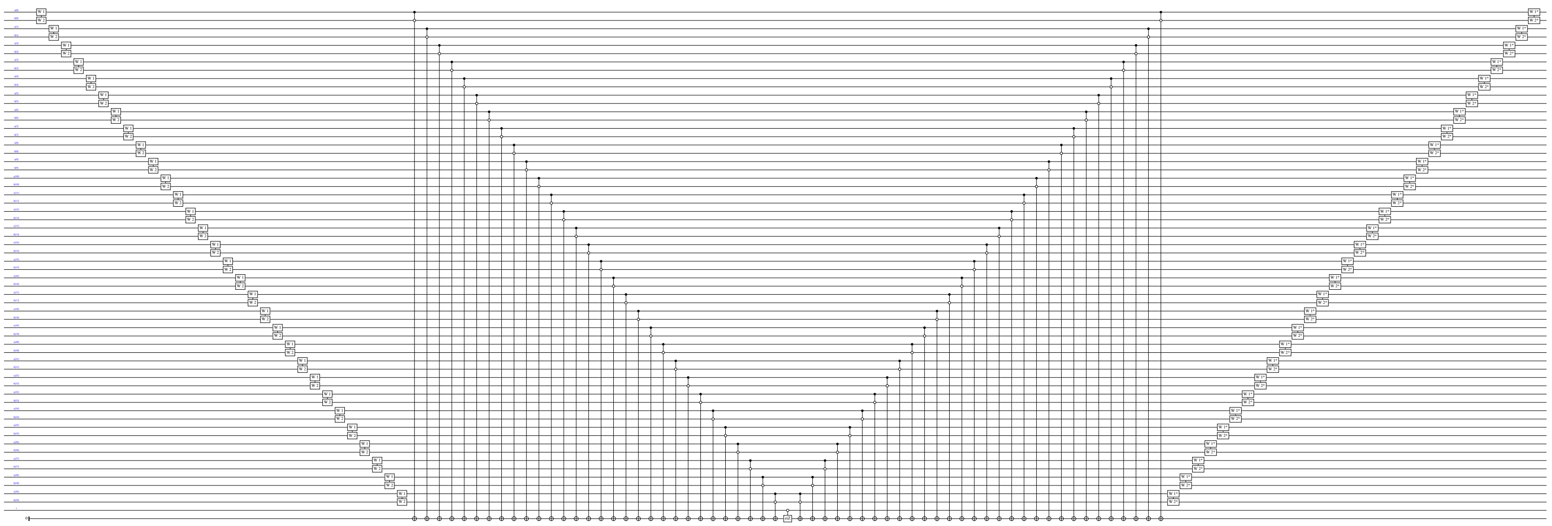
Coding quantum algorithms: Quipper

Result (3 wires):



Coding quantum algorithms: Quipper

Result (30 wires):



Coding quantum algorithms: Quipper

Built on Haskell's static type system, but

- unchecked linearity

```
controlled (qnot x) x
```

- uncaught shape mismatches
  - Consider `f :: [Qubit] -> Circ [Qubit]`
  - Assume that `length (f l) = 2 * length l`
  - Then `reverse f` cannot be applied on lists of odd lengths

Towards tools for program analysis

One cannot “read” the quantum memory

- Testing / debugging expensive
- Probabilistic model
- What does it mean to have a “correct” implementation?

Emulation of circuits

- Only for “small” instances
- Taming the testing problem
- For experimentation of error models

Formal methods

- Type systems: capture errors at compile-times
- Static analyis tools: analyze quantum programs
- Proof assistants: verify code transformation and optimization

Towards a quantum compiler

Current quantum programming languages maps to quantum circuits

- native representation structures of quantum algorithms
- Good enough for visualization, numerical emulation
- But very rigid:
  - accounts for one computational model...
  - ...but misses other models
  - occults intrinsic parallelism of computation
  - fails to capture geometrical properties of backends
    $\rightarrow$ Grid-like physical layout, graph-states, *etc.*
  - Ad-hoc graphical notation

Towards a quantum compiler

A missing piece in a compilation stack

High-level	Quipper, Liqui$|\rangle$, Project-Q...
	Circuits
	QASM
	Missing IR
	Error-corrected qubits
Hardware	Physical, noisy qubits

Plan

1. Quantum memory
2. Quantum / Classical interaction
3. Internals of algorithms
4. Coding quantum algorithms
5. The language Quipper
6. A formalization : Proto-Quipper
7. Conclusion

Proto-Quipper

A core subset of Quipper [Ross 2015]: A lambda-calculus

- Focused on the circuit-description part of the language
  $\rightarrow$ no measurement
- Simple linear type system

$$
\begin{array}{lcl}
A, B & ::= & \mathsf{qubit} \mid 1 \mid A \otimes B \mid \mathsf{bool} \mid A \multimap B \mid !A \mid \mathsf{Circ}(T, U) \\
T, U & ::= & \mathsf{qubit} \mid 1 \mid T \otimes U
\end{array}
$$

- A special class of values for representing circuits
  $\rightarrow$ Built on an algebra of circuits
- Built-in circuit operators

$$
\begin{array}{lcl}
\mathsf{box} & : & !(T \multimap U) \multimap !\mathsf{Circ}(T, U) \\
\mathsf{unbox} & : & \mathsf{Circ}(T, U) \multimap !(T \multimap U) \\
\mathsf{rev} & : & \mathsf{Circ}(U, T) \multimap !\mathsf{Circ}(T, U)
\end{array}
$$

Proto-Quipper

Circuits in Proto-Quipper

Formalized as a pair $(\mathcal{S}, \mathcal{Q})$ of enumerable sets

- $\mathcal{S}$: set of circuit states
- $\mathcal{Q}$: set of wire identifiers
- Operators relating them :

$$
\begin{array}{ll}
\texttt{New} : \mathcal{P}_f(\mathcal{Q}) \to \mathcal{S} & \qquad \texttt{In} : \mathcal{S} \to \mathcal{P}_f(\mathcal{Q}) \\
\texttt{Rev} : \mathcal{S} \to \mathcal{S} & \qquad \texttt{Out} : \mathcal{S} \to \mathcal{P}_f(\mathcal{Q}) \\
\multicolumn{2}{l}{\texttt{Append} : \mathcal{S} \times \mathcal{S} \times \mathsf{Bij}_f(\mathcal{Q}) \hookrightarrow \mathcal{S} \times \mathsf{Bij}_f(\mathcal{Q})}
\end{array}
$$

- Various equations, such as

$$\texttt{In} \circ \texttt{Rev} = \texttt{Out}, \qquad \texttt{In} \circ \texttt{New} = \texttt{Out} \circ \texttt{New} = \mathsf{id}$$

Proto-Quipper

Circuits versus functions

- A value of type $U \multimap T$ is a suspended computation
- A value of type $\mathsf{Circ}(T, U)$ is a circuit and corresponds to an element of $\mathcal{S}$.

In particular

- One can access the "content" of a circuit
- The term operator rev $=$ algebra operator `Rev`
- Unboxing a circuit $=$ "running it" $=$ using `Append`

In a sense

- $\mathsf{Circ}(T, U)$ is the type for precomputed, first-order functions on quantum data
- whereas $T \multimap U$ could contain e.g. non-terminating functions

Proto-Quipper

Linear type system

- quantum data is non-duplicable
- Subtyping relation : "A duplicable element can be used only once"

$$!A <: A$$

An opaque type for qubits

- no constructors
- only accessible through circuit combinators
- or as variables

Absence of inductive types

- Only one possible shape of value for a given first-order type

$$\text{qubit} \otimes \text{bool} \qquad\qquad \text{qubit} \otimes (\text{qubit} \otimes \text{qubit})$$

Limitations of Proto-Quipper

Absence of lists or other inductive types

- Good : unboxing sends Circ(T, U) to total functions $T \multimap U$
- Bad : An element of type Circ(T, U) is one circuit
  - No representation of families of circuits, as in Quipper

Adding lists

- Makes [qubit] $\multimap$ [qubit] represent families of circuits
  (Note: not monadic. . .)
- But Circ$([\text{qubit}], [\text{qubit}])$
  - is still one circuit of $\mathcal{S}$ with a fixed number of wires
  - ruins the totality of unboxing and reversing
  - makes boxing not ill-defined : which circuit from the family ?

Mitigating Limitations of Proto-Quipper

To mitigate problems with lists: Two main solutions

1 (not ours) – Use of dependent types

- Types to correctly specify box, unbox and rev
- Burden of proof of correctness on the programmer
- Require a full first-order linear logic

2 (ours) – Only extend type system with a notion of shape

- Captures the structure of a value of first-order type
- Boxing now takes as arguments
  - A function $!(T \multimap U)$
  - A shape for T
- Does not solve the run-time error with unbox and rev
  - Allow run-time errors related to shapes (and only those)
  - Leave proof of correctness to auxiliary tool
- Joint work on this topic between LRI and CEA/Nano-Innov

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Quantum @ LRI

Thematics

- Formal methods (Benoît Valiron, Chantal Keller, Thibault Balabonski)
- Scientific computing and HPC (Benoît Valiron, Marc Baboulin)

Students

- Timothée Goubault de Brugière : Thèse CIFRE/Atos
  $\rightarrow$ Synthesis of unitaries : Householder decomposition, BFGS
- Dong-Ho Lee : Thèse CEA (just starting)
  $\rightarrow$ Formalization of Quipper-like languages

Projects

- ANR SoftQPro, European project Quantex
- Partnership with CEA-Nano-Innov, Atos/Bull, LORIA (Nancy)

Postdocs

- We have funding for at least 2 one-year postdocs!

Quantum @ LRI

We have funding for postdocs!